# Felippa’s Gauss Integration Rules for Triangles

## Triangle Gauss Quadrature Module

(Felippa: IFEM, Chapter 24 Page 24-7)

Note: we return the weight divided by 2 here whereas Felippa defines $J = \frac{1}{2}\det(\boldsymbol{J})$ (see, e.g., text just after Equation 24.32)

In[1]:=
```
TrigGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{zeta, p = rule, i = point, g1, g2, info = {{Null, Null, Null}, 0}},
   If[p == 1, info = {{1/3, 1/3, 1/3}, 1}];
   If[p == 3, info = {{1, 1, 1}/6, 1/3}; info[[1, i]] = 2/3];
   If[p == -3, info = {{1, 1, 1}/2, 1/3}; info[[1, i]] = 0];
   If[p == 6, g1 = (8 - Sqrt[10] + Sqrt[38 - 44 * Sqrt[2/5]])/18;
    g2 = (8 - Sqrt[10] - Sqrt[38 - 44 * Sqrt[2/5]])/18;
    If[i < 4, info = {{g1, g1, g1}, (620 + Sqrt[213125 - 53320 * Sqrt[10]])/3720};
     info[[1, i]] = 1 - 2 * g1];
    If[i > 3, info = {{g2, g2, g2}, (620 - Sqrt[213125 - 53320 * Sqrt[10]])/3720};
     info[[1, i - 3]] = 1 - 2 * g2]];
   If[p == 7, g1 = (6 - Sqrt[15])/21; g2 = (6 + Sqrt[15])/21;
    If[i < 4, info = {{g1, g1, g1}, (155 - Sqrt[15])/1200};
     info[[1, i]] = 1 - 2 * g1];
    If[i > 3 && i < 7, info = {{g2, g2, g2}, (155 + Sqrt[15])/1200};
     info[[1, i - 3]] = 1 - 2 * g2];
    If[i == 7, info = {{1/3, 1/3, 1/3}, 9/40}]];
   info[[2]] = info[[2]]/2;
   (* we include the division by 2 directly here *)
   If[numer, Return[N[info, 20]], Return[Simplify[info]]];];
```

# Results

In[2]:=
```
ToRSTW[info_] := Module[
    {transformMat, sumAndRst, zetas, sum, r, s, w},
    {zetas, w} = info;
    transformMat = {(* See Felippa's Equation 15.9 *)
      {1, 1, 1},
      {0, 1, 0}, (* x1=0, x2=1, x3=0 *)
      {0, 0, 1}}; (* y1=0, y2=0, y3=1 *)
    sumAndRst = transformMat.zetas;
    {sum, r, s} = sumAndRst;
    Return[{r, s, 0, w}];];
```

## Rule 1

In[3]:=
```
Table[ToRSTW[TrigGaussRuleInfo[{1, False}, i]], {i, 1}]
```

Out[3]= $\left\{\left\{\frac{1}{3}, \frac{1}{3}, 0, \frac{1}{2}\right\}\right\}$

## Rule 3

In[4]:=
```
Table[ToRSTW[TrigGaussRuleInfo[{3, False}, i]], {i, 3}]
```

Out[4]= $\left\{\left\{\frac{1}{6}, \frac{1}{6}, 0, \frac{1}{6}\right\}, \left\{\frac{2}{3}, \frac{1}{6}, 0, \frac{1}{6}\right\}, \left\{\frac{1}{6}, \frac{2}{3}, 0, \frac{1}{6}\right\}\right\}$

## Rule -3

In[5]:=
```
Table[ToRSTW[TrigGaussRuleInfo[{-3, False}, i]], {i, 3}]
```

Out[5]= $\left\{\left\{\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{6}\right\}, \left\{0, \frac{1}{2}, 0, \frac{1}{6}\right\}, \left\{\frac{1}{2}, 0, 0, \frac{1}{6}\right\}\right\}$

## Rule 6

In[6]:=
```
Table[ToRSTW[TrigGaussRuleInfo[{6, True}, i]], {i, 6}]
```

Out[6]=
```
{{0.44594849091596488632, 0.44594849091596488632, 0, 0.11169079483900573285},
 {0.10810301816807022736, 0.44594849091596488632, 0, 0.11169079483900573285},
 {0.44594849091596488632, 0.10810301816807022736, 0, 0.11169079483900573285},
 {0.09157621350977074346, 0.09157621350977074346, 0, 0.05497587182766093381},
 {0.81684757298045851308, 0.09157621350977074346, 0, 0.05497587182766093381},
 {0.09157621350977074346, 0.81684757298045851308, 0, 0.05497587182766093381}}
```

## Rule 7

In[7]:=
```
Table[ToRSTW[TrigGaussRuleInfo[{7, True}, i]], {i, 7}]
```

Out[7]=
```
{{0.10128650732345633880, 0.10128650732345633880, 0, 0.06296959027241357629 8},
 {0.79742698535308732240, 0.10128650732345633880, 0, 0.06296959027241357629 8},
 {0.10128650732345633880, 0.79742698535308732240, 0, 0.06296959027241357629 8},
 {0.47014206410511508977, 0.47014206410511508977, 0, 0.06619707639425309036 9},
 {0.05971587178976982045 9, 0.47014206410511508977, 0, 0.06619707639425309036 9},
 {0.47014206410511508977, 0.05971587178976982045 9, 0, 0.06619707639425309036 9},
 {0.33333333333333333333, 0.33333333333333333333, 0, 0.11250000000000000000}}
```