Shape functions of Hex8

Coordinates of points in the reference space

In[1]:=
```
x = {(*1*){-1, -1, -1}, (*2*){1, -1, -1}, (*3*){1, 1, -1}, (*4*){-1, 1, -1},
    (*5*){-1, -1, 1}, (*6*){1, -1, 1}, (*7*){1, 1, 1}, (*8*){-1, 1, 1}};
```

Shape functions

In[2]:=
$$\text{ShapeFunc}[n\_] := \frac{1}{8}\,(1 + r\,x[[n,\,1]])\,(1 + s\,x[[n,\,2]])\,(1 + t\,x[[n,\,3]])$$

In[3]:=
```
AllShapeFunc = Table[ShapeFunc[m], {m, 8}];
```

In[4]:=
```
AllShapeFunc // MatrixForm
```

Out[4]//MatrixForm=

$$\begin{pmatrix} \frac{1}{8}(1-r)(1-s)(1-t) \\ \frac{1}{8}(1+r)(1-s)(1-t) \\ \frac{1}{8}(1+r)(1+s)(1-t) \\ \frac{1}{8}(1-r)(1+s)(1-t) \\ \frac{1}{8}(1-r)(1-s)(1+t) \\ \frac{1}{8}(1+r)(1-s)(1+t) \\ \frac{1}{8}(1+r)(1+s)(1+t) \\ \frac{1}{8}(1-r)(1+s)(1+t) \end{pmatrix}$$

In[5]:=
```
substitution = {
    (1 - r) → rm, (1 - s) → sm, (1 - t) → tm,
    (1 + r) → rp, (1 + s) → sp, (1 + t) → tp
  };
```

In[6]:=
```
interp = AllShapeFunc /. substitution;
Print[CForm[interp]]
```

```
List((rm*sm*tm)/8.,(rp*sm*tm)/8.,(rp*sp*tm)/8.,(rm*sp*tm)/8.,(rm*sm*tp)/8.,(rp*sm*tp)/8.,(rp*sp*tp)/8.,(
  rm*sp*tp)/8.)
```

Derivative of shape functions

In[8]:=
```
AllDerivShapeWrtR = D[AllShapeFunc, r];
AllDerivShapeWrtS = D[AllShapeFunc, s];
AllDerivShapeWrtT = D[AllShapeFunc, t];
```

In[11]:=
```
AllDerivShapeWrtR // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{8}(1-s)(1-t) \\ \frac{1}{8}(1-s)(1-t) \\ \frac{1}{8}(1+s)(1-t) \\ -\frac{1}{8}(1+s)(1-t) \\ -\frac{1}{8}(1-s)(1+t) \\ \frac{1}{8}(1-s)(1+t) \\ \frac{1}{8}(1+s)(1+t) \\ -\frac{1}{8}(1+s)(1+t) \end{pmatrix}$$

In[12]:=
```
AllDerivShapeWrtS // MatrixForm
```

Out[12]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{8}(1-r)(1-t) \\ -\frac{1}{8}(1+r)(1-t) \\ \frac{1}{8}(1+r)(1-t) \\ \frac{1}{8}(1-r)(1-t) \\ -\frac{1}{8}(1-r)(1+t) \\ -\frac{1}{8}(1+r)(1+t) \\ \frac{1}{8}(1+r)(1+t) \\ \frac{1}{8}(1-r)(1+t) \end{pmatrix}$$

In[13]:=
```
AllDerivShapeWrtT // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{8}(1-r)(1-s) \\ -\frac{1}{8}(1+r)(1-s) \\ -\frac{1}{8}(1+r)(1+s) \\ -\frac{1}{8}(1-r)(1+s) \\ \frac{1}{8}(1-r)(1-s) \\ \frac{1}{8}(1+r)(1-s) \\ \frac{1}{8}(1+r)(1+s) \\ \frac{1}{8}(1-r)(1+s) \end{pmatrix}$$

In[14]:=
```
derivR = AllDerivShapeWrtR /. substitution;
Print[CForm[derivR]]
```

```
List(-0.125*(sm*tm),(sm*tm)/8.,(sp*tm)/8.,-0.125*(sp*tm),-0.125*(sm*tp),(sm*tp)/8.,(sp*tp)/8.,-0.125*(
  sp*tp))
```

In[16]:=
```
derivS = AllDerivShapeWrtS /. substitution;
Print[CForm[derivS]]
```

```
List(-0.125*(rm*tm),-0.125*(rp*tm),(rp*tm)/8.,(rm*tm)/8.,-0.125*(rm*tp),-0.125*(rp*tp),(rp*tp)/8.,(rm*
  tp)/8.)
```

In[18]:=
```
derivT = AllDerivShapeWrtT /. substitution;
Print[CForm[derivT]]
```

```
List(-0.125*(rm*sm),-0.125*(rp*sm),-0.125*(rp*sp),-0.125*(rm*sp),(rm*sm)/8.,(rp*sm)/8.,(rp*sp)/8.,(rm*
  sp)/8.)
```