hyperbeam FEE Jones matrix polarisations

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Foreword

Additional information can be found the following links:

- https://github.com/JLBLine/polarisation_tests_for_FEE
- https://cira-pulsars-and-transients-group.github.io/vcsbeam/definitions.html

1 The reference paper

The reference paper "Calibration and Stokes Imaging with Full Embedded Element Primary Beam Model for the Murchison Widefield Array" defines its FEE Jones matrices the following way:

$$\begin{bmatrix} J_{x\theta} & J_{x\phi} \\ J_{y\theta} & J_{j\phi} \end{bmatrix}$$

where

- x is the instrument's east-west dipole;
- y is the instrument's north-south dipole;
- θ is the zenith angle; and
- ϕ is the azimuth angle, increasing clockwise from north through east.

The notation $J_{x\theta}$ should be thought of as "goes from θ to x".

2 hyperbeam

hyperbeam has always derived Jones matrices in the above "format". However, we often want to convert the "local sky" (θ, ϕ) basis to a "celestial sky" (x, y) one; this conversion is called the parallactic-angle correction. But along with this conversion, we have to be extremely careful how we define x and y sky coordinates.

The International Astronomical Union (IAU) convention is to define celestial-sky x as the north-south polarisation, and celestial-sky y as the east-west polarisation. Unfortunately, the MWA routinely defines its x and y coordinates in the opposite way (i.e. like the dipoles on the ground). To get around this confusion, in this document, we will use RA to refer to the celestial east-west-aligned polarisation, and Dec as the celestial north-south-aligned polarisation.

3 How hyperbeam applies the parallactic-angle correction

- Obtains a FEE beam-response Jones matrix $B = \begin{bmatrix} J_{x\theta} & J_{x\phi} \\ J_{y\theta} & J_{y\phi} \end{bmatrix}$;
  - Remember, x and y here are instrumental and refer to east-west and north-south, respectively.
- Re-orders B into $B' = \begin{bmatrix} -J_{y\phi} & J_{y\theta} \\ -J_{x\phi} & J_{x\theta} \end{bmatrix}$;
- Calculates a parallactic angle χ with the ERFA function eraHd2pa;
  - To do this, it needs an Earth latitude. If this is not provided, no parallactic-angle correction is done.
- Rotates B' by $\chi+\frac{\pi}{2}$, i.e. calculates $B'' = B'P$ where $P = \begin{bmatrix} \cos\left(\chi+\frac{\pi}{2}\right) & -\sin\left(\chi+\frac{\pi}{2}\right) \\ \sin\left(\chi+\frac{\pi}{2}\right) & \cos\left(\chi+\frac{\pi}{2}\right) \end{bmatrix} = \begin{bmatrix} -\sin\chi & -\cos\chi \\ \cos\chi & -\sin\chi \end{bmatrix}$;
  - $B'' = \begin{bmatrix} -B'_{10}\cos\chi + B'_{11}\sin\chi & -B'_{10}\sin\chi - B'_{11}\cos\chi \\ -B'_{00}\cos\chi + B'_{01}\sin\chi & -B'_{00}\sin\chi - B'_{01}\cos\chi \end{bmatrix}$, using $B' = \begin{bmatrix} B'_{00} & B'_{01} \\ B'_{10} & B'_{11} \end{bmatrix}$
- If an "IAU compliant" Jones matrix is desired (i.e. $\begin{bmatrix} J_{x\,Dec} & J_{x\,RA} \\ J_{y\,Dec} & J_{y\,RA} \end{bmatrix}$), then B'' is returned.
- If an "MWA compliant" Jones matrix is desired (i.e. $\begin{bmatrix} J_{x\,RA} & J_{x\,Dec} \\ J_{y\,RA} & J_{y\,Dec} \end{bmatrix}$), then $B''' = \begin{bmatrix} B''_{11} & B''_{10} \\ B''_{01} & B''_{00} \end{bmatrix}$ is returned.

An "IAU compliant" Jones matrix is obtained by specifying "iau_order" as True or 1. An "MWA compliant" Jones matrix is obtained by specifying "iau_order" as False or 0.

As stated above, if no Earth latitude is provided, then no parallactic-angle correction is done. If no parallactic-angle correction is done, then hyperbeam will return B, regardless of what "iau_order" is.

4 Versions

hyperbeam version 0.4.0 could apply a parallactic-angle correction, however:

1. The Earth latitude was hard-coded to the MWA latitude;
2. The returned Jones matrix was always in the "IAU order".