Eclipse computations in nyx

Christopher Rabotin

November 7, 2019

This computation assumes that all objects are spherical.

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1 Summary

In short, we project the light source onto a plane which crosses the eclipsing body perpendicularly to the direction between the spacecraft and the (potentially) eclipsing body. We then find whether the disks representing the eclipsing body and the light source on that plane overlap. If they do not, then we're in full illumination. If they do, we compute the overlapping area of both disks, and compute the nominal apparent disk. The ratio of these areas is used to compute the percentage of penumbra.

2 Derivation

2.1 Umbra or visibilis computation

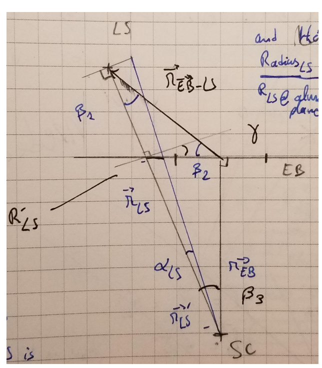

Figure 1: Top-view of eclipsing problem

First, let's compute β_2 the angle between $\vec{r}_{\text{EB-LS}}$ and $\vec{r}_{\text{LS}}$, respectively the vector from the eclipsing body to the light source and the vector from the spacecraft to the light source. If that angle is less than a right angle, then the light source is not behind the eclipsing body, so we're in full illumination: *visibilis*.

Then, we compute β_3 the angle between the spacecraft and the eclipsing body, and between the spacecraft and the light source. We need this to project the radius of the light source onto the plane centered at the eclipsing geoid, and normal to the direction to the spacecraft.

Using the triangle formed between the spacecraft, the center of the eclipsing body, and the projection plane, we can compute $\vec{r'}_{\text{LS}}$, the vector from the spacecraft to the intersection point on the plane in the direction of $\hat{\boldsymbol{r}}_{\text{LS}}$. We use β_3 for this computation as we know the length of that hypotenuse is

$$|\vec{r}_{\text{EB-LS}}| = |\vec{r'}_{\text{LS}}| \cos \beta_3$$

Using Thales' theorem, we can compute the "pseudo light source radius", that is the radius of the light source as seen from an angle γ from the plane. Forming a triangle between the intersection point, the center of the eclipsing body, and the orthogonal projection of that intersection point onto $\vec{r}_{\text{EB-LS}}$, we can compute the actual radius of the light source on the projection plane.

Let $\vec{r}_{\text{Plane-LS}}$ be the vector from the center of the eclipsing body to the projected center of the light source on the plane. We now check for any overlap. If the norm of $\vec{r}_{\text{Plane-LS}}$ minus the project radius is greater than the radius of the eclipsing body, it means that, no matter what the direction is, the shaddow of the eclipsing body *ends* before the closest point of the light source: the light source is fully visible. If the norm of $\vec{r}_{\text{Plane-LS}}$ plus the projected radius of the light source is less than the radius of the eclipsing body, then the light source is fully behind the eclipsing body, so we're in total eclipse (*umbra*). Note that we have ruled out the light source being in front of the eclipsing body by computing the angle β_2 at the start.

2.2 Penumbra percentage

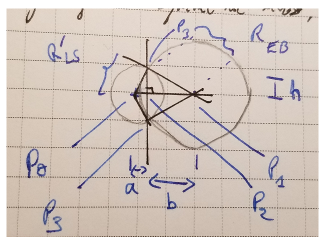

Figure 2: In plane view

Both circles represent the light source and the eclipsing body. We then use the Circle-Circle intersection computation to compute the area of the asymmetric lens corresponding to the overlap of both circles, A_{shadow}. Then, we compute the full area of the light source, $\mathcal{A}_{\text{LS}}$.

The penumbra value, P, is such that a number close to one means that the light source is almost in full visibility. Conversely, if the number is close to zero, then we are near total umbra.

$$P = \frac{\mathcal{A}_{\text{LS}} - \mathcal{A}_{\text{shadow}}}{\mathcal{A}_{\text{LS}}}$$