Valera's question

Question: Quickly find a function $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ through the points

$$(0.1, 0.2), (0.5, 0.5), (1, 0.7), (2, 0.8), (5, 0.8), (10, 1)$$

such that $f|_{[10,\infty)}$ is approximately constant.

The answer depends on "what you expect" from the "function", and how general the answer should be.

Obviously the easiest thing to do would be define some discontinuous function which satisfies the requirements and gives random values elsewhere, but I don't imagine this will have much utility.

Most likely you wish for the function to be continuous and differentiable. A very famous approach would be to interpolate your desired points using splines:

- Linear interpolation, or polygonal curve, would just "connect the dots" to form a function. The nice thing about this is that for a given left step size h_l, right step size h_r and apex p you could express the basis functions

$$\phi_{h_l,h_r,p}(x) = \begin{cases} \frac{1}{h_l}(x - p - h_l), & p - h_l < x < p \\ \frac{1}{h_r}(p + h_r - x), & p < x < p + h_r \\ 0, & \text{else} \end{cases}$$

  as compile time expressions, and very easily have the compiler generate your functions at compile time for you.
- Cubic spline interpolation is much more popular because it produces splines which have continuous first and second derivatives. This means that they look pretty smooth, they have a well-defined length, which depends continuously on the inteval length, and you can easily find their tangent. Even though determining the smooth cubic spline through fixed nodes requires solving a linear system of equations, since the system turns out to be tridiagonal and diagonally dominated, one can apply the simpler Thomas' algorithm to solve the system in $O(n)$ time. Check out the following references:

  https://www.researchgate.net/publication/338853028_Fast_Cubic_Spline_Interpolation

  https://docs.rs/cubic_spline/latest/cubic_spline/

If you wish for a closed formula involving only elementary functions, this is much more difficult. However it seems like you're looking for a monotonously increasing function that approaches 1. Consider either some shift/scaling of $-1/x$ or arctan If you're willing to accept piecewise defined functions, think about some variant of e^{-x^2}.

Though the general approach for finding a closed formula is something like this: Write the most general formula you're willing to accept with all parameters. Say that your parameters are $p_1, \ldots, p_n$. Your formula is then some function $F(p_1, \ldots, p_n; x)$. Consider the error function:

$$\varepsilon(p_1, \ldots, p_n) = \|F(p_1, \ldots, p_n; x) - f(x)\|$$

This is a nonnegative function, and the task of finding the best formula amounts to minimizing this function as a function of the parameters. Since you have finitely many points, this minimization problem boils down to a problem of solving a possibly overspecified (non-)linear system of equations, for which a common approach is the non-linear least squares algorithm. https://en.wikipedia.org/wiki/Non-linear_least_squares This approach is not "fast" though.